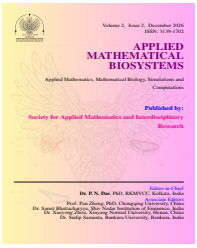
Applied Mathematical Biosystems, Volume 2, Issue 2, (Jul-Dec 2026), 63-70

Applied Mathematical Biosystems

journal homepage: https://ambjournal.com



Some Aspects of Applications of Selected Mathematical Methods to Scientific and Engineering Problems

Sukhamoy Pattanayak[1], Adhir Baran Chattopadhyay[2], Arnab Roy[3], Sumit Kanjilal[4] and Subhajyoti Das[5]

[1]*Department of Mathematics, Budge Budge Institute of Technology, Budge Budge, Kolkata-700138, India*
[2]*Department of Electrical Engineering, Budge Budge Institute of Technology, Budge Budge, Kolkata-700138, India*
[3]*Department of Physics, Budge Budge Institute of Technology, Budge Budge, Kolkata-700138, India*
[4]*Department of Civil Engineering, Budge Budge Institute of Technology, Budge Budge, Kolkata-700138, India*
[5]*Department of Computer Science & Engineering, Budge Budge Institute of Technology, Budge Budge, Kolkata-700138, India*

Abstract

Application-based learning provides a framework for connecting scientific and mathematical principles with physical and engineering problems. The present paper examines two applications of mathematical methods with this instructional motivation. The first investigates the use of the unilateral Laplace transform for one-dimensional quantum-well problems, with particular emphasis on the role of boundary conditions.For the infinite square well, the Laplace-transform formulation recovers the conventional quantised energy spectrum without introducing an additional physical boundary condition. For the tilted infinite well, a formal transform-domain equation is obtained and treated using an integrating factor; transform convergence, inverse transformation and complete spectral analysis are outside the present scope. The second application develops an analytical approximation for the geometrical arc-length increment of a deflected beam segment. Using the Euler–Bernoulli deflection function for a clamped–clamped beam carrying a concentrated static load at one-third of its span, the calculation is restricted to the segment between the left clamp and the load point. A small-slope binomial expansion is used to derive the analytical approximation, whose accuracy is assessed by comparison with numerical quadrature. Although the examples arise from different disciplines, they illustrate a common modelling process involving governing equations, boundary conditions, analytical procedures and assessment of assumptions and limitations. Their potential contribution to application-based engineering education is proposed rather than established through learning-outcome assessment.

Keywords: *Laplace transform; Schrödinger equation; infinite square well; tilted potential well; Euler–Bernoulli beam; arc-length approximation; interdisciplinary modelling*

Corresponding author: Sukhamoy Pattanayak *E-mail address:* sukhamoypattnayak@bbit.edu.in
Received: June 23, 2026 **Revised:** October 30, 2026 **Accepted:** November 4, 2026 **Published:** November 13, 2026
© Jul-Dec 2026 Society for Applied Mathematics and Interdisciplinary Research **DOI:** 10.67029/j.amb.2026.0022.25
Article classification: Research Article; **MSC 2020: 97M50 · 44A10 · 74K10 · 81Q05**

1. Introduction

1.1. Mathematical Methods in Engineering Education

Mathematics plays a fundamental role in engineering education by providing the analytical tools required to formulate, interpret and solve physical problems. Concepts such as differential equations, integral transforms and approximation methods are particularly important because they connect theoretical principles with practical engineering applications. However, the effective learning of these methods requires more than the memorisation of standard formulae and procedures. Their application to physical problems can help students understand the underlying assumptions, mathematical structure and limitations of the models they employ [1].

Problem-based learning provides an opportunity to strengthen the connection between mathematical theory and engineering practice [1–3]. By applying familiar mathematical techniques to problems drawn from different areas of physics and engineering, students can explore the relationship between mathematical procedures and physical interpretation. The present paper follows this instructional motivation through examples involving quantum-well equations and the geometrical analysis of a deflected beam. Related engineering applications of integral transforms have been discussed by Kanjilal and Chattopadhyay [4].

1.2. Laplace Transform and Quantum-Well Problems

The time-independent Schrödinger equation is a fundamental differential equation in quantum mechanics. Its solutions depend not only on the potential-energy function but also on

the spatial domain and the boundary conditions imposed by the physical system. Quantum-well problems provide useful examples for examining the relationship between differential equations, boundary conditions and the quantisation of energy [5]. Fourier-transform methods also have an established role in quantum mechanics, particularly in relating position-space and momentum-space representations of a wavefunction [5, 6].

In the infinite square well (ISW), the particle is confined to a finite region by infinitely high potential barriers. The wavefunction must satisfy the corresponding endpoint conditions, leading to a discrete energy spectrum. In the tilted infinite square well (TSW), a linearly varying interior potential, such as that associated with a uniform electric field, modifies the governing differential equation and provides a more involved mathematical problem [7].

The Laplace transform is widely used in differential-equation analysis. It converts derivatives with respect to the original variable into algebraic expressions involving the transformed function and initial data. Other operations, such as multiplication by the original variable, can introduce differentiation with respect to the transform variable [8]. Consequently, the resulting transform-domain equation may be algebraic or differential, depending on the structure of the original equation.

Its application to quantum-mechanical problems is not proposed here as a new method of solving the Schrödinger equation. Laplace-transform approaches to quantum-mechanical differential equations have a substantial history. Englefield considered the solution of the Schrödinger equation by Laplace transformation [9], while Tsaur and Wang developed a transform-based treatment of several solvable models [10]. Riahi subsequently applied Laplace transformation to time-dependent Schrödinger evolution [11]. Related work has also examined Laplace-type integral methods for energy-eigenvalue problems [12].

More recent analysis has identified methodological problems in some earlier transform treatments of stationary quantum equations, particularly in the handling of boundary conditions and inverse transformation [13]. These contributions differ in their equations, transform constructions and treatment of boundary or initial data. The present study examines the transform as a complementary analytical and pedagogical procedure for selected quantum-well problems.

For the ISW, the transform-domain formulation is developed, and the inverse transform is used to recover the interior spatial solution. The left-wall condition is incorporated during transformation, while the condition at the opposite wall subsequently determines the admissible energy values. Particular attention is given to the relationship between the transform-domain solution, the recovered wavefunction and the endpoint conditions on the finite spatial interval. Any mathematical continuation of the interior solution beyond $x = L$ is not interpreted as the physical wavefunction outside the infinite well.

For the TSW, the formal transform procedure produces a first-order linear ordinary differential equation in the transform variable. An integrating-factor representation is obtained. As in the ISW discussion, no physical meaning is assigned to a mathematical continuation beyond the right-hand wall. Convergence of the unilateral transform, inverse transformation and complete spectral analysis are not established in the present work.

1.3. Arc-Length Analysis of Deflected Beams

The deformation of structural members under transverse loading is a standard topic in engineering mechanics. Within the linear Euler–Bernoulli model, the transverse deflection depends on the loading, support conditions and flexural rigidity of the beam [20]. In addition to the transverse deflection itself, the geometry of the deformed configuration can be characterised by the arc length of the beam's centreline [14]. If the transverse deflection is represented by $y(x)$, the arc length over an interval $x_1 \le x \le x_2$ is

$$S[y] = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx. \tag{1}$$

Arc length is a standard functional encountered in the calculus of variations [14, 15]. In the present work, the deflection curve is supplied by the beam model, and this functional is evaluated along that curve. The purpose is not to derive the beam equilibrium equation by making a functional stationary.

Analytical and numerical beam methods provide a broader context for this geometrical calculation. Laplace-transform and decomposition techniques have been applied to cantilever deformation [16], while closed-form approaches have also been developed for non-uniform Euler–Bernoulli beams and frames [17]. These studies address different configurations and objectives; they are cited as related structural-analysis methods rather than as sources of the arc-length result derived here.

The structural example considered in this paper consists of a uniform beam clamped at both ends and subjected to a concentrated static load at $x = L/3$, where L is the span. The arc-length calculation is restricted to the segment $0 \le x \le L/3$, extending from the left clamp to the point of load application. Its arc length is

$$S_1 = \int_0^{L/3} \sqrt{1 + \left[y_1'(x)\right]^2}\, dx, \tag{2}$$

where $y_1(x)$ denotes the deflection on that segment. The corresponding geometrical increment relative to the undeformed projected length is

$$\Delta S_1 = S_1 - \frac{L}{3}. \tag{3}$$

The resulting integral is not generally convenient to evaluate in elementary form. A small-slope binomial expansion is therefore used to obtain an analytical approximation. Its truncation accuracy is assessed by comparison with numerical quadrature of the corresponding arc-length integral. This comparison concerns the approximation for the adopted curve; it does not independently validate the physical beam model or identify the calculated geometrical increment with material axial elongation.

1.4. Scope and Organisation of the Study

This study examines selected applications of mathematical methods in quantum mechanics and structural engineering. For the infinite square well, the Laplace-transform formulation recovers the standard energy spectrum and normalized eigenfunctions. The tilted-well analysis is restricted to a formal transform-domain equation and its integrating-factor representation. Transform convergence, inverse transformation and complete spectral analysis of the tilted-well problem are outside the present scope.

The structural analysis evaluates the arc-length increment of a selected segment of a clamped beam's deflection curve. A truncated binomial expansion provides an analytical

approximation, whose accuracy is assessed by comparison with numerical quadrature. This assessment concerns the approximation of the arc length for the prescribed curve, rather than independent validation of the beam model or determination of material axial elongation. The total centreline-length increment of the complete beam is not calculated.

The paper proceeds from the mathematical formulation of the quantum-well problems and the beam model to the corresponding transform-domain analyses and arc-length approximation. The results are then discussed with emphasis on the mathematical assumptions, limitations and potential educational value of the examples. These applications are presented as instructional illustrations; the study does not claim to establish their educational effectiveness.

2. Problem Formulations

This section presents the mathematical formulation of the two problems considered in this paper. The first concerns a quantum particle confined in an infinite potential well, both in the absence and in the presence of a uniform electric field. The second concerns the deflection of a clamped beam subjected to a concentrated transverse load and the determination of the arc length of a selected portion of its deflected configuration. Although the two applications describe different physical systems, both require a governing differential equation, clearly defined parameters, and appropriate boundary conditions. An explicit statement of the mathematical foundations for the analysis is presented in the following subsections.

2.1. Quantum-Mechanical Problems: Governing Equation and Boundary Conditions

The stationary states of a quantum particle moving in a one-dimensional potential field are governed by the time-independent Schrödinger equation,

$$-\frac{\hbar^2}{2m}\frac{d^2\phi(x)}{dx^2} + V(x)\phi(x) = E\phi(x), \tag{4}$$

where $\phi(x)$ denotes the spatial wavefunction, m is the mass of the particle, $\hbar$ is the reduced Planck constant, $V(x)$ is the potential energy and E is an energy parameter whose admissible eigenvalues are determined by the potential, spatial domain, and boundary conditions.

For an infinite potential well extending over the interval $0 < x < L$, the wavefunction must vanish at both boundaries. Within the well, the Schrödinger equation is an ordinary differential equation whose solution must satisfy these boundary conditions. The derivative at the left endpoint is not independently prescribed by the first wall condition. It is therefore retained symbolically as an undetermined constant during the transform calculation. The Laplace-transform approach is used below to formulate the corresponding differential equations in the transform domain. Particular attention is given to the role of the boundary condition at the origin and the subsequent imposition of the condition at the opposite wall. For this elementary problem, the direct characteristic-equation method is shorter; the Laplace-transform treatment is presented as a complementary analytical and instructional procedure.

2.1.1. Laplace-Transform Solution of the Infinite Square Well

Consider a particle confined in an infinite square well (ISW) of width L, described by the potential

$$V(x) = \begin{cases} 0, & 0 < x < L, \\ \infty, & x \le 0 \text{ or } x \ge L. \end{cases} \tag{5}$$

Within the well, the time-independent Schrödinger equation is

$$-\frac{\hbar^2}{2m}\phi''(x) = E\phi(x), \qquad 0 < x < L. \tag{6}$$

Defining the spectral parameter

$$\kappa^2 = \frac{2mE}{\hbar^2}, \tag{7}$$

Eq. (6) becomes

$$\phi''(x) + \kappa^2\phi(x) = 0. \tag{8}$$

The infinite potential barriers impose the boundary conditions

$$\phi(0) = 0, \qquad \phi(L) = 0. \tag{9}$$

The inverse-transformed solution is used on the physical interval $0 \le x \le L$, and the boundary condition at $x = L$ is subsequently imposed to determine the admissible eigenvalues [8] . Any mathematical continuation of the interior solution beyond $x = L$ is not interpreted as the physical wavefunction outside the infinite well.

Let $\Phi(s) = \mathcal{L}\{\phi(x)\}$ denote the unilateral Laplace transform, where s is the transform variable and e^{-sx} is the transform kernel. Using the derivative rule

$$\mathcal{L}\{\phi''(x)\} = s^2\Phi(s) - s\phi(0) - \phi'(0), \tag{10}$$

the Laplace transform of Eq. (8) gives

$$(s^2 + \kappa^2)\Phi(s) = s\phi(0) + \phi'(0). \tag{11}$$

The first boundary condition, $\phi(0) = 0$, reduces this expression to

$$(s^2 + \kappa^2)\Phi(s) = C, \qquad C = \phi'(0), \tag{12}$$

where C is an undetermined constant. For a nontrivial solution, $C \neq 0$. Hence,

$$\Phi(s) = \frac{C}{s^2 + \kappa^2}. \tag{13}$$

Taking the inverse Laplace transform yields

$$\phi(x) = \frac{C}{\kappa}\sin(\kappa x). \tag{14}$$

The second boundary condition, $\phi(L) = 0$, then requires

$$\sin(\kappa L) = 0. \tag{15}$$

Excluding the trivial solution, the allowed values of the spectral parameter are

$$\kappa_n = \frac{n\pi}{L}, \qquad n = 1, 2, 3, \ldots. \tag{16}$$

Using Eq. (7), the corresponding energy eigenvalues are

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}. \tag{17}$$

Normalization requires

$$\int_0^L |\phi_n(x)|^2\, dx = 1. \tag{18}$$

Using the inverse-transformed solution

$$\phi_n(x) = \frac{C_n}{\kappa_n}\sin(\kappa_n x),$$

the normalization condition determines the magnitude of the integration constant as

$$|C_n| = \kappa_n\sqrt{\frac{2}{L}}.$$

Its phase, however, remains arbitrary. Thus, the integration constant can be expressed as

$$C_n = \kappa_n\sqrt{\frac{2}{L}}e^{i\theta},$$

where θ is an arbitrary real phase. The normalized eigenfunction can therefore be written as

$$\phi_n(x) = e^{i\theta}\sqrt{\frac{2}{L}}\sin\Big(\frac{n\pi x}{L}\Big). \tag{19}$$

The conventional choice $\theta = 0$ gives the real-valued representation

$$\phi_n(x) = \sqrt{\frac{2}{L}}\sin\Big(\frac{n\pi x}{L}\Big). \tag{20}$$

Other choices, including $\theta = \pi/2$, π and $3\pi/2$, yield integration constants proportional to $+i\kappa_n$, $-\kappa_n$ and $-i\kappa_n$, respectively. These choices correspond to different global phases of the same normalized state and do not change its probability density or other physical predictions [5]. Importantly, these possibilities follow directly from the mathematical formulation without requiring a prior interpretation in terms of momentum eigenstates.

The same spatial solution admits an equivalent representation in terms of complex exponentials,

$$\sin(\kappa_n x) = \frac{e^{i\kappa_n x} - e^{-i\kappa_n x}}{2i}. \tag{21}$$

This representation expresses the sine function as a linear combination of exponential components with opposite wave numbers. Although the sine eigenfunction is not itself a momentum eigenfunction, its complex-exponential representation provides a mathematical connection to the description of momentum components. This connection can be explored after obtaining the solution directly from the differential equation and its boundary conditions, without requiring such an interpretation as a starting assumption.

The corresponding probability density is

$$|\phi_n(x)|^2 = \frac{2}{L}\sin^2\Big(\frac{n\pi x}{L}\Big). \tag{22}$$

Thus, the transform-domain formulation, followed by inverse transformation and application of the second boundary condition, recovers the standard energy spectrum and normalized eigenfunctions of the infinite square well. In this context it is worth mentioning, that the sine eigenfunction is not itself a momentum eigenfunction, this decomposition provides a mathematical starting point for discussing momentum components and alternative representations of quantum states. It is important to distinguish this decomposition from the arbitrary global phase: changing the global phase does not generate additional momentum components. Rather, the exponential representation makes the two opposite wave-number contributions explicit and offers a basis for further investigation of momentum representations and related applications.

2.1.2. Transform-Domain Formulation for the Tilted Infinite Well

Consider a tilted infinite square well (TSW) described by the potential

$$V(x) = \begin{cases} 2bx, & 0 < x < L, \\ \infty, & x \le 0 \text{ or } x \ge L, \end{cases} \tag{23}$$

where b is a constant with dimensions of energy per unit length. The linear potential represents the effect of a uniform electric field within the well, with the sign of b determined by the particle charge and the coordinate convention.

Within the well, the time-independent Schrödinger equation is

$$-\frac{\hbar^2}{2m}\phi''(x) + 2bx\phi(x) = E\phi(x). \tag{24}$$

Rearranging gives

$$\phi''(x) + (\kappa^2 - \gamma x)\phi(x) = 0, \tag{25}$$

where

$$\kappa^2 = \frac{2mE}{\hbar^2}, \qquad \gamma = \frac{4mb}{\hbar^2}. \tag{26}$$

The corresponding units are

$$[b] = \mathrm{J\,m^{-1}}, \qquad [\kappa^2] = \mathrm{m^{-2}}, \qquad [\gamma] = \mathrm{m^{-3}}. \tag{27}$$

Thus, κ^2 and γx have the same units. Here κ^2 denotes the spectral coefficient $2mE/\hbar^2$; this notation does not impose an additional positivity restriction for every possible sign of b. The infinite barriers impose the boundary conditions

$$\phi(0) = 0, \qquad \phi(L) = 0. \tag{28}$$

Let $\Phi(s) = \mathcal{L}\{\phi(x)\}$ denote the unilateral Laplace transform. Applying the derivative rule and using $\phi(0) = 0$, the transform of Eq. (25) gives

$$(s^2 + \kappa^2)\Phi(s) - \phi'(0) - \gamma\mathcal{L}\{x\phi(x)\} = 0. \tag{29}$$

Using the identity

$$\mathcal{L}\{x\phi(x)\} = -\frac{d\Phi(s)}{ds}, \tag{30}$$

the transformed equation becomes

$$\gamma\frac{d\Phi(s)}{ds} + (s^2 + \kappa^2)\Phi(s) = C, \qquad C = \phi'(0). \tag{31}$$

Unlike the ISW equation, Eq. (31) is a first-order linear differential equation in the transform variable s. This result illustrates an important limitation of the elementary Laplace-transform approach: a linear spatial potential produces differentiation with respect to the transform variable, rather than a purely algebraic equation. A complete eigenvalue analysis of the TSW, including determination of the allowed energies from both hard-wall boundary conditions, is beyond the present elementary transform treatment. Nevertheless, the transformed function can be obtained formally using an integrating factor [18, 19]. For $\gamma \neq 0$, its integrating factor is

$$\mu(s) = \exp\left(\frac{s^3}{3\gamma} + \frac{\kappa^2 s}{\gamma}\right). \tag{32}$$

Multiplying Eq. (31) by $\mu(s)/\gamma$ gives

$$\frac{d}{ds}\left[\mu(s)\Phi(s)\right] = \frac{C}{\gamma}\mu(s). \tag{33}$$

Integrating with respect to s, the general solution of the transform-domain differential equation can be expressed as

$$\Phi(s) = \frac{1}{\mu(s)}\left[D + \frac{C}{\gamma}\int_{s_0}^{s} \mu(u)\,du\right], \tag{34}$$

where D is an integration constant and s_0 is a reference point. The integration constant can formally be written as $D = \mu(s_0)\Phi(s_0)$. Its admissible value must, however, be consistent with the defining properties of the unilateral Laplace transform and the prescribed initial data; it cannot be selected independently of those requirements.These compatibility and convergence conditions are not established in the present formal treatment.

The transform-domain solution in Eq. (34) establishes a first-order formulation for the tilted-well problem. The inverse transformation and complete determination of the TSW energy spectrum are not pursued in the present work. The second boundary condition, $\phi(L) = 0$, would have to be imposed on the recovered spatial solution to determine the admissible energy eigenvalues. Accordingly, the present analysis is restricted to the transform-domain equation and its integrating-factor solution.

2.2. Arc-Length Formulation for the Deflected Beam

The second application concerns the geometrical arc length of a selected segment of a beam after transverse deflection. The purpose is not to propose a new beam-deflection theory, but to evaluate an arc-length functional using a specified deflection curve.

Consider a uniform Euler–Bernoulli beam clamped at both end. The beam has constant flexural rigidity EI, where E is Young's modulus and I is the second moment of area of the cross-section about the bending axis. Its initial span is L, and a concentrated downward load of magnitude $W > 0$ acts at $x = L/3$. The longitudinal coordinate x is measured from the left clamp, and $y(x)$ denotes transverse displacement, taken as positive upward.

The beam is modelled as slender, linearly elastic, and prismatic, with constant flexural rigidity EI. Shear deformation is neglected, and the transverse deflection is obtained from the linear Euler–Bernoulli model[20] .

The notation used in this subsection is:

- E: Young's modulus of the beam material, with units N m^{-2};
- I: second moment of area of the cross-section about the bending axis, with units m^4;
- EI: flexural rigidity, with units N m^2;
- W: magnitude of the applied point force, with units N;
- L, x, and y: quantities with units of length.

Here E denotes Young's modulus, not the energy parameter used in Section 2.1.

The support conditions are

$$y(0) = y'(0) = y(L) = y'(L) = 0. \tag{35}$$

Using the sign convention adopted here, the governing equation is

$$EI\,y^{(4)}(x) = -W\delta\left(x - \frac{L}{3}\right), \tag{36}$$

where $\delta(\cdot)$ denotes the Dirac delta distribution. The concentrated load produces a jump in the third derivative of the deflection, while the deflection, slope and second derivative remain continuous at the load application point[21, 22].

The resulting piecewise deflection is

$$y_1(x) = -\frac{2Wx^2(3L - 5x)}{81EI}, \qquad 0 \le x \le \frac{L}{3}, \tag{37}$$

and

$$y_2(x) = -\frac{2Wx^2(3L - 5x)}{81EI} - \frac{W}{6EI}\left(x - \frac{L}{3}\right)^3, \qquad \frac{L}{3} \le x \le L. \tag{38}$$

The second expression can equivalently be written as

$$y_2(x) = -\frac{W(x - L)^2(7x - L)}{162EI}. \tag{39}$$

Differentiating the deflection functions gives the slopes

$$y_1'(x) = \frac{2Wx(5x - 2L)}{27EI}, \qquad 0 \le x \le \frac{L}{3}, \tag{40}$$

and

$$y_2'(x) = -\frac{W(x - L)(7x - 3L)}{54EI}, \qquad \frac{L}{3} \le x \le L. \tag{41}$$

These expressions satisfy the clamped support conditions and the required continuity and concentrated-load conditions at $x = L/3$.

2.2.1. Approximate Arc-Length Increment of the Deflected Beam Segment ($0 \le x \le L/3$)

The present analysis is restricted to the beam segment $0 \le x \le L/3$, extending from the left support to the point of application of the concentrated load. For a deflection function $y_1(x)$, the arc length of this segment is

$$S_1 = \int_0^{L/3} \sqrt{1 + \left[y_1'(x)\right]^2}\,dx. \tag{42}$$

The corresponding arc-length increment relative to the undeformed projected length $L/3$ is

$$\Delta S_1 = S_1 - \frac{L}{3}. \tag{43}$$

For sufficiently small slopes, the binomial expansion

$$\sqrt{1 + u} = 1 + \frac{u}{2} - \frac{u^2}{8} + \frac{u^3}{16} + \cdots \tag{44}$$

can be applied with $u = [y_1'(x)]^2$. The following approximation assumes that the magnitude of the beam slope remains small throughout the selected segment, $0 \le x \le L/3$. Under this assumption, successive powers of the squared slope contribute progressively smaller corrections to the arc-length integral, permitting truncation of the binomial expansion. The approximation should not be used outside this regime without assessing the neglected higher-order contributions. Retaining the first two nonzero terms in the arc-length increment gives

$$\Delta S_1 \simeq \frac{1}{2}\int_0^{L/3} [y_1'(x)]^2\,dx - \frac{1}{8}\int_0^{L/3} [y_1'(x)]^4\,dx. \tag{45}$$

Substitution of Eq. (40) and evaluation of the integrals yield

$$\Delta S_1 \simeq \frac{4W^2L^5}{177147E^2I^2} - \frac{3196W^4L^9}{3295011258945E^4I^4}. \tag{46}$$

For convenience, introduce the dimensionless parameter

$$\lambda = \frac{WL^2}{EI} \tag{47}$$

and the dimensionless coordinate $t = x/L$. The slope can then be expressed as

$$y_1'(x) = \lambda f(t), \qquad f(t) = \frac{2t(5t-2)}{27}, \qquad 0 \le t \le \frac{1}{3}. \tag{48}$$

Consequently, Eq. (46) becomes

$$\frac{\Delta S_1}{L} \simeq \frac{4\lambda^2}{177147} - \frac{3196\lambda^4}{3295011258945}. \tag{49}$$

Numerical comparison and error assessment. To assess the accuracy of the analytical approximation, the arc-length integral is also evaluated numerically over the same interval. In dimensionless form, the numerical reference value is

$$\frac{\Delta S_{1,\mathrm{num}}}{L} = \int_0^{1/3} \sqrt{1+\lambda^2 f^2(t)}\,dt - \frac{1}{3}. \tag{50}$$

The relative error is calculated as

$$\varepsilon_{\mathrm{rel}} = \frac{\left|\Delta S_{1,\mathrm{approx}} - \Delta S_{1,\mathrm{num}}\right|}{\Delta S_{1,\mathrm{num}}} \times 100\%. \tag{51}$$

Table 1 presents the comparison for representative values of the dimensionless parameter λ.

Table 1. Numerical and analytical arc-length increments for the beam segment $0 \le x \le L/3$.

λ	$\Delta S_{1,\mathrm{num}}/L$	$\Delta S_{1,\mathrm{approx}}/L$	Relative error (%)
1	$2.25791472 \times 10^{-5}$	$2.25791471 \times 10^{-5}$	3.982×10^{-7}
5	$5.63898108 \times 10^{-4}$	$5.63896707 \times 10^{-4}$	2.484×10^{-4}
10	$2.24840104 \times 10^{-3}$	$2.24831220 \times 10^{-3}$	0.003951
20	$8.88234573 \times 10^{-3}$	$8.87685465 \times 10^{-3}$	0.06182

The comparison indicates that the analytical approximation agrees closely with the numerical evaluation for the parameter values considered. The relative error increases with λ, as expected when higher-order terms in the binomial expansion become more significant. The approximation should therefore be used only when the small-slope assumption remains appropriate. The numerical comparison provides an assessment for the values of the selected parameters rather than a universal error bound. Close agreement with numerical quadrature does not, by itself, establish the validity of the underlying linear beam model[23]. The small-slope and other modelling assumptions must therefore be checked separately for any physical application.

3. Educational Contribution and Instructional Relevance

The examples considered in this study share a common modelling sequence: formulate the governing differential equation, identify the relevant parameters and their dimensions, select an appropriate mathematical procedure, impose the physical boundary conditions, and assess the resulting solution. This sequence provides opportunities to connect mathematical methods with physical modelling across quantum mechanics and structural analysis.

For the infinite square well, the Laplace transform provides an alternative route to solving the interior differential equation, while the endpoint conditions determine the allowed energy spectrum. The solution also illustrates the distinction between mathematical formulation and physical interpretation: the phase of the integration constant remains arbitrary after normalization, and the resulting sine function admits an equivalent representation in terms of complex exponentials with opposite wave numbers. This provides an opportunity to discuss alternative mathematical representations of a quantum state without presupposing a momentum-space description. For the tilted infinite well, the position-dependent potential introduces a derivative with respect to the transform variable, illustrating how the structure of a transformed equation changes with the form of the potential. For the beam problem, the deflection function provides the basis for calculating the geometrical arc length, while the binomial approximation and numerical comparison illustrate the assessment of approximation accuracy.

These examples are proposed as instructional material for illustrating connections between mathematical methods and physical modelling; they do not constitute evidence of improved student learning outcomes. Their pedagogical effectiveness could be investigated in a future educational study using structured classroom activities, pre- and post-instruction assessments, and a comparison of conventional and transform-based solution approaches.

3.1. Proposed Classroom Activity

A classroom activity based on the present examples may be organised around the following tasks:

1. Derive the transform-domain equation for the infinite square well (ISW), retaining the unknown initial derivative as an undetermined constant, and explain how the boundary condition at the opposite wall determines the allowed energy levels.
2. Verify the transform relation for $x\phi(x)$ and compare the mathematical structures of the ISW and tilted infinite square well (TSW) transform-domain equations.
3. Differentiate the beam-deflection function for the first segment, express the resulting slope in dimensionless form, and derive the leading-order approximation to the arc-length increment.
4. Compare the truncated arc-length approximation with numerical quadrature and assess the approximation error over the segment.

The intended learning outcomes include the ability to distinguish initial data from physical boundary conditions, verify dimensional consistency, identify the scope and limitations of a transform-domain calculation, and distinguish approximation error from physical modelling error. These outcomes are proposed for future assessment and have not been evaluated in the present

study.

4. Conclusions

The principal findings and implications of the present work are summarised below.

1. *Application of the Laplace transform to the infinite square well.* The unilateral Laplace transform provides an alternative analytical route to the conventional solution of the Schrödinger equation for the infinite square well. Treating the interior differential equation as an initial-value problem, followed by imposition of the physical boundary condition at the opposite wall, recovers the familiar quantised energy spectrum. The derivative at the initial endpoint enters the transformed equation as an initially unspecified constant; it does not constitute an additional physical boundary condition. The analysis also illustrates the role of the arbitrary global phase and the equivalent representation of the sine eigenfunction in terms of complex exponentials with opposite wave numbers. This representation provides a mathematical starting point for discussing momentum components without implying that the present analysis establishes additional physical momentum states. The contribution therefore lies in the presentation of the solution procedure and its instructional potential rather than in a new quantum-mechanical result.
2. *Transform-domain formulation for the tilted infinite square well.* The tilted-well example illustrates how a position-dependent potential changes the structure of the transformed equation by introducing differentiation with respect to the transform variable. The resulting first-order linear equation is treated using an integrating factor. This analysis is restricted to a formal transform-domain formulation: convergence of the unilateral Laplace transform, inverse transformation and application of the right-wall condition to determine the energy spectrum are not established. The example therefore demonstrates the structure of the transformed problem rather than a completed solution of the tilted-well eigenvalue problem.
3. *Analytical estimation of beam-segment arc length.* For a clamped–clamped beam subjected to a concentrated static load at$x = L/3$, the adopted deflection function is used to evaluate the geometrical arc length of the segment $0 \leq x \leq L/3$. A truncated binomial expansion yields an analytical approximation for the arc-length increment of this segment. Its truncation accuracy is assessed by comparison with numerical quadrature, while its physical applicability remains subject to the small-slope assumption and the underlying beam-model assumptions. The segment is selected to simplify the mathematical demonstration rather than to restrict the applicability of the procedure. With an appropriate extension of the integration domain and use of the corresponding deflection function, the same approach could be applied to estimate the geometrical arc-length increment over the entire beam centreline. This extension would provide a basis for evaluating the total geometrical length change, although this quantity would not, by itself, represent the material axial elongation.
4. *Potential instructional relevance and further applications.* The quantum-mechanical and beam examples illustrate a common modelling workflow: specify the governing equations and relevant conditions, apply an analytical procedure, and examine the scope and limitations of the resulting solution. They may therefore serve as instructional examples connecting mathematical methods with physical and engineering problems. In the infinite-square-well example, the complex-exponential representation may motivate further investigation of transform-based methods for problems involving multiple momentum components. Such an extension would require a separate analysis of the relevant momentum representation and physical boundary conditions and remains beyond the scope of the present work. Although the examples have potential instructional value, the present study does not establish improved student learning outcomes. The beam arc-length procedure could be extended to other static loading configurations, including triangular and uniformly distributed loads, provided that the corresponding deflection functions and approximation conditions are established. Further extensions to impulsive loading, including a sequence or train of impulses, and to blast loading would require a dynamic beam formulation accounting for inertia and, where appropriate, damping.

Overall, the contribution of this work is the presentation and examination of selected analytical procedures, with explicit attention to their assumptions, boundary conditions, and limitations. The examples are intended to support interdisciplinary mathematical modelling rather than to establish new fundamental results in quantum mechanics or structural mechanics.

Declarations

Acknowledgment: The authors would like to thank the Reviewers for their valuable comments and constructive suggestions, which have helped in improving the quality and clarity of the manuscript. The authors also acknowledge Dr. K. Nandi for his assistance with LaTeX formatting.

Funding: The author did not receive any kind financial support from any organization for this study. Author also has no financial or non-financial interests to disclose.

Data Availability: No external datasets were used or generated during the current study. All results are based on the mathematical analysis and numerical simulations presented in the manuscript.

Conflict of Interest: The author declares that there is no conflict of interest regarding the publication of this article.

Ethical Approval: Ethical approval was not applicable for this study.

Use of AI Tools: The authors declare that AI tools were used for language editing to improve the clarity and readability of the manuscript. However, the authors take full responsibility for the accuracy, scientific integrity, and originality of the content.

Author Contributions: Dr. A. B. Chattopadhyay: Conceptualization, selection of the research problems, formulation of the mathematical model and overall objectives. Dr. A. Roy: Mathematical calculations and insightful analysis of the underlying physical principles and implications of the quantum-mechanical formulation. Mr. S. Kanjilal: Mathematical

calculations and analysis of the arc length of the deflected beam. Dr. S. Pattanayak: Development of mathematical arguments to justify the proposed models, particularly by establishing the conditions for the applicability of the Laplace transform and determining its region of convergence (ROC) under the boundary conditions associated with two different forms of potential; development of the dimensionless representation; and assessment of the validity of the approximate results through comparison with numerical quadrature. Mr. S. Das: Generation of numerical data and computational support for the numerical quadrature analysis.

REFERENCES

[1] B. Pepin, R. Biehler, and G. Gueudet, "Mathematics in engineering education: A review of the recent literature with a view towards innovative practices," *International Journal of Research in Undergraduate Mathematics Education*, vol. 7, no. 2, pp. 163–188, 2021. doi: 10.1007/s40753-021-00139-8.

[2] P. Thakur, S. Dutt, and A. Chauhan, "Problem Based Learning in Education—Its Need for Sustainable Development," *Journal of Engineering Education Transformations*, vol. 34, no. 4, pp. 58–66, 2021. doi: 10.16920/jeet/2021/v34i4/152993.

[3] M. J. Prince and R. M. Felder, "Inductive teaching and learning methods: Definitions, comparisons, and research bases," *Journal of Engineering Education*, vol. 95, no. 2, pp. 123–138, 2006. doi: 10.1002/j.2168-9830.2006.tb00884.x.

[4] S. Kanjilal and A. B. Chattopadhyay, "Application of Discontinuous Mathematical Functions to Engineering Problems through Integral Transforms: Case Studies," in *Emerging Trends in Smart Computing, Communication and Automation*, 1st ed., edited by H. Biswas, J. S. Banerjee, S. Bhattacharyya, D. De, P. Sarigiannidis, J. Platoš, and M. M. Asad, CRC Press, London, United Kingdom, 2026, Chapter 25, pp. 110–113. doi: 10.1201/9781042026333-25.

[5] D. J. Griffiths and D. F. Schroeter, *Introduction to Quantum Mechanics*, 3rd ed. Cambridge University Press, 2018. doi: 10.1017/9781316995433.

[6] D. T. Si, "The Fourier Transform and Principles of Quantum Mechanics," *Applied Mathematics*, vol. 9, no. 4, pp. 347–354, 2018. doi: 10.4236/am.2018.94026.

[7] J. N. Churchill and F. O. Arntz, "The Infinite Tilted-Well: An Example of Elementary Quantum Mechanics with Applications toward Current Research," *American Journal of Physics*, vol. 37, no. 7, pp. 693–697, 1969. doi: 10.1119/1.1975775.

[8] J. L. Schiff, *The Laplace Transform: Theory and Applications*. Springer, New York, 1999. doi: 10.1007/978-0-387-22757-3.

[9] M. J. Englefield, "Solution of Schrödinger equation by Laplace transform," *Journal of the Australian Mathematical Society*, vol. 8, no. 3, pp. 557–567, 1968. doi: 10.1017/S1446788700006212.

[10] G.-Y. Tsaur and J. Wang, "A universal Laplace-transform approach to solving Schrödinger equations for all known solvable models," *European Journal of Physics*, vol. 35, no. 1, article 015006, 2014. doi: 10.1088/0143-0807/35/1/015006.

[11] N. Riahi, "Solving the time-dependent Schrödinger equation via Laplace transform," *Quantum Studies: Mathematics and Foundations*, vol. 4, no. 2, pp. 103–126, 2017. doi: 10.1007/s40509-016-0087-5.

[12] J. Canfield, A. Galler, and J. K. Freericks, "The Laplace method for energy eigenvalue problems in quantum mechanics," *Quantum Reports*, vol. 5, no. 2, pp. 370–397, 2023. doi: 10.3390/quantum5020024.

[13] L. M. Báez and A. Santos, "Revisiting the Laplace transform in quantum mechanics: correcting a flawed approach for the stationary Schrödinger equation," *European Journal of Physics*, vol. 46, no. 2, article 025401, 2025. doi: 10.1088/1361-6404/adafed.

[14] M. L. Boas, *Mathematical Methods in the Physical Sciences*, 3rd ed. John Wiley & Sons, 2006.

[15] I. M. Gelfand and S. V. Fomin, *Calculus of Variations*, translated and edited by R. A. Silverman. Dover Publications, 2000.

[16] R. Theinchai, S. Chankan, and W. Yukunthorn, "Application of ADM using Laplace transform to approximate solutions of nonlinear deformation for cantilever beam," *International Journal of Mathematics and Mathematical Sciences*, vol. 2016, article 5052194, 2016. doi: 10.1155/2016/5052194.

[17] J. C. Molina-Villegas, J. E. Ballesteros Ortega, and G. Martínez Martínez, "Closed-form solution for non-uniform Euler–Bernoulli beams and frames," *Engineering Structures*, vol. 292, article 116381, 2023. doi: 10.1016/j.engstruct.2023.116381.

[18] G. F. Simmons, *Differential Equations with Applications and Historical Notes*, with a chapter by J. S. Robertson, Tata McGraw-Hill, New Delhi, 2003, pp. 502–537.

[19] M. Hermann and M. Saravi, *A First Course in Ordinary Differential Equations: Analytical and Numerical Methods*. Springer, 2014. doi: 10.1007/978-81-322-1835-7.

[20] A. F. Bower, *Applied Mechanics of Solids*. CRC Press, Boca Raton, 2010.

[21] E. Chicurel-Uziel, "Unit step and impulse function equations to simplify the solution of engineering problems," *Journal of Information Sciences and Computing Technologies*, vol. 7, no. 1, pp. 654–666, 2018.

[22] A. Palmeri and A. Cicirello, "Physically-based Dirac's delta functions in the static analysis of multi-cracked Euler–Bernoulli and Timoshenko beams," *International Journal of Solids and Structures*, vol. 48, nos. 14–15, pp. 2184–2195, 2011. doi: 10.1016/j.ijsolstr.2011.03.024.

[23] L. N. Trefethen, *Approximation Theory and Approximation Practice*, extended ed. Society for Industrial and Applied Mathematics, Philadelphia, 2019. doi: 10.1137/1.9781611975949.