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# An Optimized Inventory Model for Perishable Items with Cost and Visibility-Driven Demand, Partial Backlogging, Price Escalation, and Value Adjustment in Fuzzy Environment

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**Abstract**

This paper develops an inventory model for Weibull-deteriorating items with price- and stock-dependent demand, incorporating partial backlogging under inflation and time discounting over a finite horizon. The demand rate is defined as $D(t) = \gamma - \lambda p + \gamma_1 I(t)$ when $I(t) > 0$ and $D(t) = \gamma - \lambda p$ when $I(t) \leq 0$, where $\gamma$ is the basic market demand, $\lambda$ the price sensitivity, $p$ the selling price, $\gamma_1$ the stock attraction parameter, and $I(t)$ the inventory level. Deterioration follows a two-parameter Weibull distribution $\theta(t) = \alpha_d \beta_d t^{\beta_d - 1}$, and shortages are partially backlogged with rate $S(\tau) = 1/(1 + \delta\tau)$. The objective is to minimize the discounted total cost. The deterministic model is first solved and then extended to a fuzzy environment by treating $\gamma$, $\lambda$, and $\delta$ as triangular fuzzy numbers, defuzzified using the expected value method. Numerical examples and sensitivity analyses demonstrate that the fuzzy model offers greater stability under uncertainty, making it suitable for short shelf-life products such as fashion goods and electronics.

**Keywords:** *Inventory model; Weibull deterioration; Price-dependent demand; Stock-dependent demand; Partial backlogging; Inflation; Fuzzy environment.*

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## 1. Introduction

Inventory systems for deteriorating products are of critical importance in industries where items lose value over time due to spoilage, obsolescence, decay, or evaporation, such as food processing, pharmaceuticals, fashion retail, and electronic goods. Classical inventory models initially assumed constant demand and non-deteriorating items, which are rarely realistic in practical situations (Harris, 1913) [1].

In real markets, demand is influenced by several operational and marketing factors. In particular, selling price plays a significant role, as lower prices generally stimulate higher demand (Dutta and Pal, 1991) [2]. Similarly, the quantity of goods displayed affects customer perception and purchasing behavior, leading to stock-dependent demand patterns commonly observed in retail environments (Sarkar et al., 1997) [3]. Inventory models that incorporate both price- and stock-dependent demand therefore provide a more realistic representation of market dynamics (Dutta and Pal, 2001) [4].

For many products, deterioration does not occur at a constant rate but increases with time due to aging effects. The two-parameter Weibull distribution offers a flexible framework for modeling such time-dependent deterioration and has been widely adopted in deteriorating inventory systems (Covert and Philip, 1973) [5]. When inventory decisions span a long planning horizon, economic factors such as inflation and the time value of money further influence replenishment policies, making discounted cost evaluation essential (Ray and Chaudhuri, 1997; Wee and Law, 2001) [6, 7].

Shortages are often unavoidable in practice due to demand uncertainty or supply disruptions. During stock-out periods, not all customers are willing to wait for replenishment, resulting in partial backlogging and lost sales. A realistic approach assumes that the proportion of backlogged demand decreases with waiting time, which has been effectively modeled using time-dependent backlogging functions (Chang and Dye, 1999) [8].

In addition to these operational complexities, many inventory parameters—such as demand intensity, price sensitivity, and backlogging behavior—are not precisely known and are subject to managerial judgment or market uncertainty. Fuzzy set theory provides a useful framework for handling such imprecision by representing uncertain parameters as fuzzy numbers rather than fixed values (Mahata and Goswami, 2009) [9].

Motivated by these considerations, this paper develops an inventory model that integrates price- and stock-dependent demand, time-dependent Weibull deterioration, partial backlogging of shortages, and inflation within both crisp and fuzzy environments. The crisp model formulates inventory dynamics using differential equations and evaluates the present value of total relevant costs over a finite planning horizon. The fuzzy extension captures parameter uncertainty and applies the expected value method for defuzzification.

The objective of the proposed model is to determine the optimal replenishment cycle length and order quantity, allowing shortages, so as to minimize the total inventory cost. Numerical illustrations and sensitivity analyses demonstrate that the fuzzy model offers improved robustness and stability under uncertain conditions, making it particularly suitable for products with short life cycles and volatile demand patterns.

## 2. Literature Review

The development of inventory theory began with the classical Economic Order Quantity (EOQ) model assuming constant demand, as proposed by Harris (1913) [1]. Recognizing that demand patterns vary over time, Silver and Meal (1969) [10] extended EOQ models to accommodate time-dependent demand structures. More developments in fuzzy inventory modeling include the work of Jana, Das, and Maiti (2013) [11], who proposed a fuzzy EOQ model with backorders under stock-dependent demand.

Subsequent studies incorporated market-driven demand characteristics. Stock-dependent demand, reflecting the influence of displayed inventory on customer purchasing behavior, was investigated by Sarkar et al. (1997) [12]. Price-dependent demand models, capturing the inverse relationship between selling price and demand, were developed by Dutta and Pal (1991) [2]. Later, combined price- and stock-dependent demand models were proposed by Dutta and Pal (2001) [4] and Hou (2006) [13], offering more realistic representations of retail environments.

Deterioration modeling also evolved significantly. Covert and Philip (1973) [5] introduced the two-parameter Weibull distribution to represent time-dependent deterioration, providing greater flexibility than constant-rate decay models. The interaction between deterioration and financial considerations was later examined by Wee and Law (2001) [7], who incorporated inflation and the time value of money into deteriorating inventory systems.

The treatment of shortages progressed with the introduction of partial backlogging models. Chang and Dye (1999) [8] proposed a waiting-time-dependent backlogging rate, capturing customer impatience during stock-out periods. This formulation was further applied and refined by Kundu and Chakrabarti (2012) [14] in deteriorating inventory environments.

The impact of inflation and time-based discounting over a finite planning horizon was systematically studied by Ray and Chaudhuri (1997) [15], emphasizing the importance of discounted cost evaluation in long-term inventory decision-making.

To address uncertainty in system parameters, fuzzy inventory models have been widely explored. Mahata and Goswami (2009) [16] incorporated fuzziness into deteriorating inventory models by representing key parameters as fuzzy numbers. Valliappan and Uthayakumar (2010) [17] applied the signed distance method for defuzzification, while alternative approaches, such as the expected value method, have been adopted for their computational simplicity.

Despite these extensive studies, limited work has simultaneously integrated price- and stock-dependent demand, Weibull deterioration, partial backlogging, and inflation within a unified fuzzy framework. The present study aims to bridge this gap by developing a comprehensive model that addresses both operational realism and parameter uncertainty.

## 3. Assumptions and Notations

### 3.1. Assumptions

The following assumptions are adopted to develop the proposed inventory model:

1. A single deteriorating item is considered over a finite planning horizon $H$, which is divided into $m$ identical replenishment cycles, each of length
$$T = \frac{H}{m}.$$
2. The demand rate is both price-dependent and stock-dependent, and is defined as
$$D(t) = \begin{cases} \gamma - \lambda p + \gamma_1 I(t), & \text{if } I(t) > 0, \\ \gamma - \lambda p, & \text{if } I(t) \leq 0, \end{cases}$$
where $\gamma$ denotes the basic market demand, $\lambda$ is the price sensitivity parameter, $p$ is the selling price (assumed constant for simplicity), $\gamma_1$ is the stock attraction parameter, and $I(t)$ represents the inventory level at time $t$.
3. The deterioration rate follows a two-parameter Weibull distribution given by
$$\theta(t) = \alpha_d \beta_d t^{\beta_d - 1},$$
where $\alpha_d > 0$ is the scale parameter ($\alpha_d \ll 1$) and $\beta_d > 1$ is the shape parameter.
4. Shortages are permitted and partially backlogged. The backlogging rate is a decreasing function of the waiting time $\tau$ and is expressed as
$$S(\tau) = \frac{1}{1 + \delta\tau},$$
where $\delta > 0$ is the backlogging parameter. The term $1 - S(\tau)$ represents the fraction of lost sales.
5. Inflation and the time value of money are considered. The net discount rate is defined as
$$R = r - i,$$
where $r$ denotes the inflation rate and $i$ denotes the interest (or discount) rate.
6. Replenishment is instantaneous, and the lead time is assumed to be zero.
7. Deteriorated items are withdrawn immediately from inventory and are neither repaired nor replaced.
8. In the fuzzy environment, the parameters $\gamma$, $\lambda$, and $\delta$ are assumed to be triangular fuzzy numbers.

### 3.2. Notations

- $T$: The replenishment cycle.
- $T_1$: Time inventory zeros in a cycle.

- $Q$: The order quantity during a cycle of length $T$.
- $I(t)$: On-hand inventory level at time $t$ during any cycle.
- $C_o$: The ordering cost per order of the inventory per cycle.
- $C_h$: The holding cost per unit per unit time in a cycle.
- $C_d$: The deterioration cost per unit per cycle.
- $C_s$: The shortage cost per unit per unit time (backlogged).
- $C_l$: The lost sales cost per unit per cycle.
- PVTC: The present value of total cost.
- $\widehat{\text{PVTC}}$: The fuzzy present value of total cost.

## 4. Crisp Model Formulation

Suppose that when the inventory level is positive, the demand rate depends on $\gamma - \lambda p + \gamma_1 I(t)$,whereas for negative inventory levels, the demand (backlogging) rate is given by $\gamma - \lambda p$. Therefore, the no-shortage period occurs over the interval $[0, T_1]$ of the scheduling period $T$, while shortages are partially backlogged during the interval $[T_1, T]$. The behavior of this inventory system is illustrated in Fig. 1.

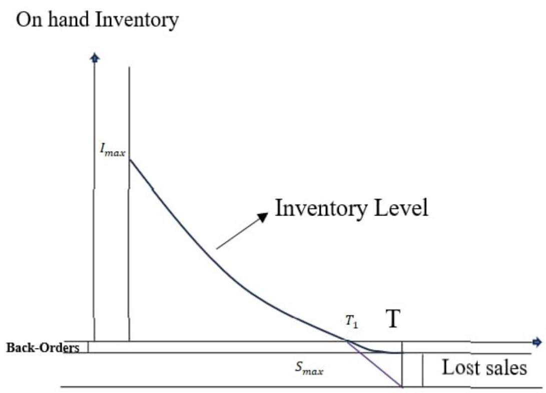


**Figure 1.** Graphical representation of the Inventory Model.

The first replenishment lot size $Q$ is received at time $t = 0$. During the time interval $[0, T_1]$, the inventory level decreases due to the combined effects of stock- and price-dependent demand and item deterioration, and reaches zero at $t = T_1$. During the time interval $[T_1, T]$, shortages occur and are accumulated until time $t = T$, at which point they are backordered. Therefore, the inventory system at any time $t$ can be represented by the following differential equations:
For $0 \leq t \leq T_1$, the inventory level satisfies

$$\frac{dI(t)}{dt} = -D(t) - \theta(t)I(t) = -(\gamma - \lambda p + \gamma_1 I(t)) - \alpha_d \beta_d t^{\beta_d - 1} I(t), \tag{1}$$

with the boundary conditions

$$I(0) = I_{\max} \quad \text{and} \quad I(T_1) = 0.$$

Using these conditions, the solution of Eq. (1) is obtained as

$$I(t) = (\gamma - \lambda p)\left[(T_1 - t) - \frac{\delta}{2}(T_1^2 - t^2) - \frac{\alpha_d \beta_d}{\beta_d + 1}(T_1^{\beta_d+1} - t^{\beta_d+1})\right], \tag{2}$$

and the maximum inventory level is given by

$$I_{\max} = (\gamma - \lambda p)\left[T_1 - \frac{\delta}{2}T_1^2 - \frac{\alpha_d \beta_d}{\beta_d + 1}T_1^{\beta_d+1}\right]. \tag{3}$$

For $T_1 \leq t \leq T$, shortages occur with partial backlogging:

During the shortage period, the rate of change of inventory is governed by

$$\frac{dI(t)}{dt} = -\frac{\gamma - \lambda p}{1 + \delta(T - t)}, \tag{4}$$

where $D(t) = \gamma - \lambda p$, $S(\tau) = \dfrac{1}{1 + \delta \tau}$, and $\tau = T - t$. Using the boundary conditions

$$I(T_1) = 0 \quad \text{and} \quad I(T) = S_{\max},$$

the solution of Eq. (4) is obtained as

$$I(t) = -(\gamma - \lambda p)\left[(t - T_1) - \delta\left\{T(t - T_1) - \left(\frac{t^2}{2} - \frac{T_1^2}{2}\right)\right\}\right], \tag{5}$$

and the maximum shortage level is given by

$$\begin{aligned} S_{\max} &= -(\gamma - \lambda p)\left[(T - T_1) - \delta\left\{T(T - T_1)\right.\right. \\ &\quad \left.\left. - \left(\frac{T^2}{2} - \frac{T_1^2}{2}\right)\right\}\right]. \end{aligned} \tag{6}$$

The lot size during the total time interval $[0, T]$ is given by

$$\begin{aligned} Q &= I_{\max} + S_{\max} \\ &= (\gamma - \lambda p)\left[T_1 - \frac{\delta}{2}T_1^2 - \frac{\alpha_d \beta_d}{\beta_d + 1}T_1^{\beta_d+1}\right] \\ &\quad -(\gamma - \lambda p)\left[(T - T_1) - \delta\left\{T(T - T_1) - \left(\frac{T^2}{2} - \frac{T_1^2}{2}\right)\right\}\right]. \end{aligned} \tag{7}$$

According to the above discussion, the following cost functions can be derived. The present value of the ordering cost is $PV_{oc} = C_o\, e^{-RT}$. The holding cost applies only when the inventory level is positive, that is, over the interval $[0, T_1]$. Therefore, the present value of the holding cost is $PV_{hc} = C_h \int_0^{T_1} I(t)e^{-Rt}\, dt$, which, upon simplification, yields

$$\begin{aligned} PV_{hc} &= C_h(\gamma - \lambda p)\left[\frac{T_1^2}{2} - \frac{\delta}{3}T_1^3 - \frac{\alpha_d \beta_d}{\beta_d + 2}T_1^{\beta_d+2}\right. \\ &\quad \left. -R\left\{\frac{T_1^3}{6} - \frac{\delta}{8}T_1^4 - \frac{\alpha_d \beta_d}{2(\beta_d + 3)}T_1^{\beta_d+3}\right\}\right]. \end{aligned} \tag{8}$$

The present value of deterioration cost is incurred only during the positive inventory phase. The number of units deteriorated at time $t$ is given by $\theta(t)I(t)$. Hence, the present value of deterioration cost per cycle is $PV_{dc} = C_d \int_0^{T_1} \theta(t)I(t)e^{-Rt}\, dt$, which, after simplification, yields

$$\begin{aligned} PV_{dc} &= C_d \alpha_d \beta_d(\gamma - \lambda p)\left[\frac{1}{\beta_d(\beta_d+1)}T_1^{\beta_d+1} - \frac{\gamma_1}{\beta_d(\beta_d+2)}T_1^{\beta_d+2}\right. \\ &\quad - \frac{\alpha_d}{2\beta_d+1}T_1^{2\beta_d+1} - R\left\{\frac{1}{(\beta_d+1)(\beta_d+2)}T_1^{\beta_d+2} - \frac{\gamma_1}{(\beta_d+1)(\beta_d+3)}T_1^{\beta_d+3}\right. \\ &\quad \left.\left. - \frac{\alpha_d \beta_d}{(\beta_d+1)(2\beta_d+2)}T_1^{2\beta_d+2}\right\}\right]. \end{aligned} \tag{9}$$

The present value of shortage cost applies to backlogged shortages during the interval $[T_1, T]$. Therefore, the present value of shortage cost is given by $PV_{sc} = C_s \int_{T_1}^{T} [-I(t)]e^{-Rt}\, dt$, which simplifies to

$$PV_{sc} = C_s(\gamma - \lambda p)\Big[\tfrac{1}{2}(T^2 - T_1^2) - T_1(T - T_1) - \delta\Big\{\tfrac{T}{2}(T^2 - T_1^2) - TT_1(T - T_1) - \tfrac{1}{6}(T^3 - T_1^3) + \tfrac{T_1^2}{2}(T - T_1)\Big\} - R\Big[\tfrac{1}{3}(T^3 - T_1^3) - \tfrac{T_1}{2}(T^2 - T_1^2) - \delta\Big\{\tfrac{T}{3}(T^3 - T_1^3) - \tfrac{TT_1}{2}(T^2 - T_1^2) - \tfrac{1}{8}(T^4 - T_1^4) + \tfrac{T_1^2}{4}(T^2 - T_1^2)\Big\}\Big]\Big]. \quad (10)$$

Lost sales occur during the shortage phase for the fraction of demand that is not backlogged. Hence, the present value of lost sales cost per cycle is expressed as $PV_{lc} = C_l \int_{T_1}^{T} D(t)\big[1 - S(T - t)\big]\, dt$, which simplifies to

$$PV_{lc} = \delta(\gamma - \lambda p)\Big[(T - \delta T^2)(T - T_1) + \frac{2\delta T - 1}{2}(T^2 - T_1^2) - \frac{\delta}{3}(T^3 - T_1^3)\Big]. \quad (11)$$

Therefore the present value total cost per cycle is, Total PVTC($T_1$)= PVOC ($PV_{oc}$) + PV Holding Cost ($PV_{hc}$) + PV Deterioration Cost ($PV_{dc}$) + PV Shortage Cost ($PV_{sc}$) + PV Lost Sales Cost ($PV_{lc}$). Hence, total PVTC($T_1$) becomes

$$\begin{aligned}
&C_o e^{-RT} + C_h(\gamma - \lambda p)\Bigg[\tfrac{T_1^2}{2} - \tfrac{\delta}{3}T_1^3 - \tfrac{\alpha_d\beta_d}{\beta_d+2}T_1^{\beta_d+2} \\
&-R\left\{\tfrac{T_1^3}{6} - \tfrac{\delta}{8}T_1^4 - \tfrac{\alpha_d\beta_d}{2(\beta_d+3)}T_1^{\beta_d+3}\right\}\Bigg] + C_d\alpha_d\beta_d(\gamma - \lambda p) \\
&\times\Bigg[\tfrac{1}{\beta_d(\beta_d+1)}T_1^{\beta_d+1} - \tfrac{\gamma_1}{\beta_d(\beta_d+2)}T_1^{\beta_d+2} - \tfrac{\alpha_d}{2\beta_d+1}T_1^{2\beta_d+1} \\
&-\tfrac{R}{\beta_d+1}\left\{\tfrac{1}{\beta_d+2}T_1^{\beta_d+2} - \tfrac{\gamma_1}{\beta_d+3}T_1^{\beta_d+3} - \tfrac{\alpha_d\beta_d}{2\beta_d+2}T_1^{2\beta_d+2}\right\}\Bigg] \\
&+C_s(\gamma - \lambda p)\Bigg[\tfrac{1}{2}(T^2 - T_1^2) - T_1(T - T_1) \\
&-\delta\left\{\tfrac{T}{2}(T^2 - T_1^2) - TT_1(T - T_1) - \tfrac{1}{6}(T^3 - T_1^3) + \tfrac{T_1^2}{2}(T - T_1)\right\} - R\Big[\tfrac{1}{3}(T^3 - T_1^3) - \tfrac{T_1}{2}(T^2 - T_1^2) \\
&-\delta\left\{\tfrac{T}{3}(T^3 - T_1^3) - \tfrac{TT_1}{2}(T^2 - T_1^2) - \tfrac{1}{8}(T^4 - T_1^4) + \tfrac{T_1^2}{4}(T^2 - T_1^2)\right\}\Big]\Bigg] + \delta(\gamma - \lambda p)\Bigg[(T - \delta T^2)(T - T_1) \\
&+\tfrac{2\delta T-1}{2}(T^2 - T_1^2) - \tfrac{\delta}{3}(T^3 - T_1^3)\Bigg]. \quad (12)
\end{aligned}$$

## 5. Fuzzy Extensions

### 5.1. Mathematical preliminaries

To defuzzify using the Graded Mean Representation method, certain definitions are required.

**Definition:** Consider the universe of discourse as $X$. A fuzzy set $\tilde{A}$ on $X$ is defined by the ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\},$$

where the membership function is $\mu_{\tilde{A}}(x) \to [0, 1]$.

**Definition:** The definition of $\alpha$-cut of $\tilde{A}$ is as follows:

$$A_\alpha = \{x : \mu_{\tilde{A}}(x) = \alpha,\ \alpha \geq 0\}.$$

**Definition:** $\tilde{A}$ is called normal if there exists $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

**Definition:** A fuzzy triangular number $\tilde{A}$ is defined by

$$\tilde{A} = (a_{11}, b_{11}, c_{11}),$$

and its membership function $\mu_{\tilde{A}}(x)$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} L^{-1}(h) = \dfrac{x - a_{11}}{b_{11} - a_{11}}, & a_{11} \leq x \leq b_{11}, \\ R^{-1}(h) = \dfrac{c_{11} - x}{c_{11} - b_{11}}, & b_{11} \leq x \leq c_{11}, \\ 0, & \text{otherwise.} \end{cases}$$

When $a_{11} = b_{11} = c_{11}$, then the fuzzy point is

$$(a_{11}, a_{11}, a_{11}) = \tilde{A}.$$

Let $F_n$ be the set of all fuzzy triangular numbers on the real numbers set $\mathbb{R}$ and is denoted by

$$F_n = \{(a_{11}, b_{11}, c_{11}) : a_{11} \leq b_{11} \leq c_{11},\ \forall\, a_{11}, b_{11}, c_{11} \in \mathbb{R}\}.$$

The $\alpha$-cut of $\tilde{A} = (a_{11}, b_{11}, c_{11}) \in F_n$, $0 \leq \alpha \leq 1$, is

$$A(\alpha) = [A_L(\alpha), A_R(\alpha)],$$

where

$$A_L(\alpha) = a_{11} + (b_{11} - a_{11})\alpha, \quad A_R(\alpha) = c_{11} - (c_{11} - b_{11})\alpha$$

are the left end point and right end point of $A(\alpha)$ respectively.

**Definition:** If $\tilde{A} = (a_{11}, b_{11}, c_{11})$ is a fuzzy triangular number, then the mean graded integration of $\tilde{A}$ is defined as

$$\begin{aligned} P(\tilde{A}) &= \frac{\int_0^1 h\left(\dfrac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh}{\int_0^1 h\, dh}, \\ &= \frac{\int_0^1 h\,[a_{11} + (b_{11} - a_{11})h + c_{11} - (c_{11} - b_{11})h]\, dh}{2\int_0^1 h\, dh}, \\ &= \frac{a_{11} + 4b_{11} + c_{11}}{6}. \end{aligned} \quad (13)$$

### 5.2. Fuzzy Model

Due to uncertainty, it is difficult to give a specified value for a specific cost parameter in the surroundings; for this reason, we have taken some of cost parameters $\gamma, \lambda, \delta$ as triangular fuzzy numbers. Here,

$$\tilde{\gamma} = (\gamma_1, \gamma_2, \gamma_3), \quad \tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3), \quad \tilde{\delta} = (\delta_1, \delta_2, \delta_3)$$

and

$$\widetilde{PVTC}(T_1) = \big(\widetilde{PVTC}_1(T_1), \widetilde{PVTC}_2(T_1), \widetilde{PVTC}_3(T_1)\big). \quad (14)$$

Now the total fuzzy cost is given by

$$
\begin{aligned}
\text{Min}\widehat{PVTC}(T_1) &= C_o e^{-RT} + C_h(\tilde{\gamma} - \tilde{\lambda} p)\Bigg[\frac{T_1^2}{2} - \frac{\tilde{\delta}}{3}T_1^3 - \frac{\alpha_d\beta_d}{\beta_d+2}T_1^{\beta_d+2} \\
&\quad - R\Bigg\{\frac{T_1^3}{6} - \frac{\tilde{\delta}}{8}T_1^4 - \frac{\alpha_d\beta_d}{2(\beta_d+3)}T_1^{\beta_d+3}\Bigg\}\Bigg] + C_d\alpha_d\beta_d(\tilde{\gamma} - \tilde{\lambda} p) \\
&\quad \times\Bigg[\frac{1}{\beta_d(\beta_d+1)}T_1^{\beta_d+1} - \frac{\gamma_1}{\beta_d(\beta_d+2)}T_1^{\beta_d+2} - \frac{\alpha_d}{2\beta_d+1}T_1^{2\beta_d+1} \\
&\quad - R\Bigg\{\frac{1}{(\beta_d+1)(\beta_d+2)}T_1^{\beta_d+2} - \frac{\gamma_1}{(\beta_d+1)(\beta_d+3)}T_1^{\beta_d+3} \\
&\quad - \frac{\alpha_d\beta_d}{(\beta_d+1)(2\beta_d+2)}T_1^{2\beta_d+2}\Bigg\}\Bigg] + C_s(\tilde{\gamma} - \tilde{\lambda} p)\Bigg[\frac{1}{2}(T^2 - T_1^2) \\
&\quad - T_1(T - T_1) - \tilde{\delta}\Bigg\{\frac{T}{2}(T^2 - T_1^2) - TT_1(T - T_1) - \frac{1}{6}(T^3 - T_1^3) \\
&\quad + \frac{T_1^2}{2}(T - T_1)\Bigg\} - R\Bigg[\frac{1}{3}(T^3 - T_1^3) - \frac{T_1}{2}(T^2 - T_1^2) - \tilde{\delta}\Bigg\{\frac{T}{3}(T^3 \\
&\quad - T_1^3) - \frac{TT_1}{2}(T^2 - T_1^2) - \frac{1}{8}(T^4 - T_1^4) + \frac{T_1^2}{4}(T^2 - T_1^2)\Bigg\}\Bigg]\Bigg] + \tilde{\delta}(\tilde{\gamma} - \tilde{\lambda} p) \\
&\quad \times\Bigg[(T - \tilde{\delta}T^2)(T - T_1) + \frac{2\tilde{\delta}T - 1}{2}(T^2 - T_1^2) - \frac{\tilde{\delta}}{3}(T^3 - T_1^3)\Bigg].
\end{aligned}
\tag{15}
$$

By graded mean representation method we defuzzify the fuzzy total cost $\widehat{PVTC}(T_1)$ and the fuzzy cost parameters $\tilde{\gamma}, \tilde{\lambda}, \tilde{\delta}$ as follows:

$$
\widehat{PVTC}(T_1) = \frac{1}{6}\left[\widehat{PVTC}_1(T_1) + 4\widehat{PVTC}_2(T_1) + \widehat{PVTC}_3(T_1)\right],
$$

where,

$$
\begin{aligned}
\widehat{PVTC}_1(T_1) &= C_o e^{-RT} + C_h(\hat{\gamma}_1 - \hat{\lambda}_1 p)\Bigg[\frac{T_1^2}{2} - \frac{\hat{\delta}_1}{3}T_1^3 - \frac{\alpha_d\beta_d}{\beta_d+2}T_1^{\beta_d+2} \\
&\quad - R\Bigg\{\frac{T_1^3}{6} - \frac{\hat{\delta}_1}{8}T_1^4 - \frac{\alpha_d\beta_d}{2(\beta_d+3)}T_1^{\beta_d+3}\Bigg\}\Bigg] + C_d\alpha_d\beta_d(\hat{\gamma}_1 - \hat{\lambda}_1 p) \\
&\quad \times\Bigg[\frac{1}{\beta_d(\beta_d+1)}T_1^{\beta_d+1} - \frac{\gamma_1}{\beta_d(\beta_d+2)}T_1^{\beta_d+2} - \frac{\alpha_d}{2\beta_d+1}T_1^{2\beta_d+1} \\
&\quad - \frac{R}{\beta_d+1}\Bigg\{\frac{1}{\beta_d+2}T_1^{\beta_d+2} - \frac{\gamma_1}{\beta_d+3}T_1^{\beta_d+3} - \frac{\alpha_d\beta_d}{2\beta_d+2}T_1^{2\beta_d+2}\Bigg\}\Bigg] \\
&\quad + C_s(\hat{\gamma}_1 - \hat{\lambda}_1 p)\Bigg[\frac{1}{2}(T^2 - T_1^2) - T_1(T - T_1) - \hat{\delta}_1\Bigg\{\frac{T}{2}(T^2 - T_1^2) \\
&\quad - TT_1(T - T_1) - \frac{1}{6}(T^3 - T_1^3) + \frac{T_1^2}{2}(T - T_1)\Bigg\} - R\Bigg[\frac{1}{3}(T^3 - T_1^3) \\
&\quad - \frac{T_1}{2}(T^2 - T_1^2) - \hat{\delta}_1\Bigg\{\frac{T}{3}(T^3 - T_1^3) - \frac{TT_1}{2}(T^2 - T_1^2) - \frac{1}{8}(T^4 - T_1^4) \\
&\quad + \frac{T_1^2}{4}(T^2 - T_1^2)\Bigg\}\Bigg]\Bigg] + \hat{\delta}_1(\hat{\gamma}_1 - \hat{\lambda}_1 p)\Bigg[(T - \hat{\delta}_1 T^2)(T - T_1) \\
&\quad + \frac{2\hat{\delta}_1 T - 1}{2}(T^2 - T_1^2) - \frac{\hat{\delta}_1}{3}(T^3 - T_1^3)\Bigg].
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\widehat{PVTC}_2(T_1) &= C_o e^{-RT} + C_h(\hat{\gamma}_2 - \hat{\lambda}_2 p)\Bigg[\frac{T_1^2}{2} - \frac{\hat{\delta}_2}{3}T_1^3 - \frac{\alpha_d\beta_d}{\beta_d+2}T_1^{\beta_d+2} \\
&\quad - R\Bigg\{\frac{T_1^3}{6} - \frac{\hat{\delta}_2}{8}T_1^4 - \frac{\alpha_d\beta_d}{2(\beta_d+3)}T_1^{\beta_d+3}\Bigg\}\Bigg] + C_d\alpha_d\beta_d(\hat{\gamma}_2 - \hat{\lambda}_2 p) \\
&\quad \times\Bigg[\frac{1}{\beta_d(\beta_d+1)}T_1^{\beta_d+1} - \frac{\gamma_1}{\beta_d(\beta_d+2)}T_1^{\beta_d+2} - \frac{\alpha_d}{2\beta_d+1}T_1^{2\beta_d+1} \\
&\quad - \frac{R}{\beta_d+1}\Bigg\{\frac{1}{\beta_d+2}T_1^{\beta_d+2} - \frac{\gamma_1}{\beta_d+3}T_1^{\beta_d+3} - \frac{\alpha_d\beta_d}{2\beta_d+2}T_1^{2\beta_d+2}\Bigg\}\Bigg] \\
&\quad + C_s(\hat{\gamma}_2 - \hat{\lambda}_2 p)\Bigg[\frac{1}{2}(T^2 - T_1^2) - T_1(T - T_1) \\
&\quad - \hat{\delta}_2\Bigg\{\frac{T}{2}(T^2 - T_1^2) - TT_1(T - T_1) - \frac{1}{6}(T^3 - T_1^3) + \frac{T_1^2}{2}(T - T_1)\Bigg\} \\
&\quad - R\Bigg[\frac{1}{3}(T^3 - T_1^3) - \frac{T_1}{2}(T^2 - T_1^2) \\
&\quad - \hat{\delta}_2\Bigg\{\frac{T}{3}(T^3 - T_1^3) - \frac{TT_1}{2}(T^2 - T_1^2) - \frac{1}{8}(T^4 - T_1^4) + \frac{T_1^2}{4}(T^2 - T_1^2)\Bigg\}\Bigg]\Bigg] \\
&\quad + \hat{\delta}_2(\hat{\gamma}_2 - \hat{\lambda}_2 p)\Bigg[(T - \hat{\delta}_2 T^2)(T - T_1) + \frac{2\hat{\delta}_2 T - 1}{2}(T^2 - T_1^2) - \frac{\hat{\delta}_2}{3}(T^3 - T_1^3)\Bigg].
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\widehat{PVTC}_3(T_1) &= C_o e^{-RT} + C_h(\hat{\gamma}_3 - \hat{\lambda}_3 p)\Bigg[\frac{T_1^2}{2} - \frac{\hat{\delta}_3}{3}T_1^3 - \frac{\alpha_d\beta_d}{\beta_d+2}T_1^{\beta_d+2} \\
&\quad - R\Bigg\{\frac{T_1^3}{6} - \frac{\hat{\delta}_1}{8}T_1^4 - \frac{\alpha_d\beta_d}{2(\beta_d+3)}T_1^{\beta_d+3}\Bigg\}\Bigg] + C_d\alpha_d\beta_d(\hat{\gamma}_3 - \hat{\lambda}_3 p) \\
&\quad \times\Bigg[\frac{1}{\beta_d(\beta_d+1)}T_1^{\beta_d+1} - \frac{\gamma_1}{\beta_d(\beta_d+2)}T_1^{\beta_d+2} - \frac{\alpha_d}{2\beta_d+1}T_1^{2\beta_d+1} \\
&\quad - \frac{R}{\beta_d+1}\Bigg\{\frac{1}{\beta_d+2}T_1^{\beta_d+2} - \frac{\gamma_1}{\beta_d+3}T_1^{\beta_d+3} - \frac{\alpha_d\beta_d}{2\beta_d+2}T_1^{2\beta_d+2}\Bigg\}\Bigg] \\
&\quad + C_s(\hat{\gamma}_3 - \hat{\lambda}_3 p)\Bigg[\frac{1}{2}(T^2 - T_1^2) - T_1(T - T_1) \\
&\quad - \hat{\delta}_3\Bigg\{\frac{T}{2}(T^2 - T_1^2) - TT_1(T - T_1) - \frac{1}{6}(T^3 - T_1^3) + \frac{T_1^2}{2}(T - T_1)\Bigg\} \\
&\quad - R\Bigg[\frac{1}{3}(T^3 - T_1^3) - \frac{T_1}{2}(T^2 - T_1^2) \quad - \hat{\delta}_3\Bigg\{\frac{T}{3}(T^3 - T_1^3) \\
&\quad - \frac{TT_1}{2}(T^2 - T_1^2) - \frac{1}{8}(T^4 - T_1^4) + \frac{T_1^2}{4}(T^2 - T_1^2)\Bigg\}\Bigg]\Bigg] + \hat{\delta}_3(\hat{\gamma}_3 - \hat{\lambda}_3 p) \\
&\quad \times\Bigg[(T - \hat{\delta}_3 T^2)(T - T_1) + \frac{2\hat{\delta}_3 T - 1}{2}(T^2 - T_1^2) - \frac{\hat{\delta}_3}{3}(T^3 - T_1^3)\Bigg].
\end{aligned}
\tag{18}
$$

### 5.3. Solution Procedure

To find the best value of $T_1$ and $\widehat{PVTC}(T_1)$, we must have

$$
\frac{d\,\widehat{PVTC}(T_1)}{dT_1} = 0, \qquad \frac{d^2\,\widehat{PVTC}(T_1)}{dT_1^2} > 0.
$$

### 5.4. Method for finding Total Cost in Crisp Model

Firstly, we find the total $PVTC$ in a crisp environment from equation (12). Then, applying fuzzy arithmetic on some cost parameters, also in triangular fuzzy numbers, we compute the fuzzy total $PVTC$ from equation (13). After defuzzifying using the graded mean integration method, we find the minimized total $PVTC$.

## 6. Numerical Illustration

Numerical examples are solved for both crisp and fuzzy environments to obtain the optimal $T_1$ and minimum total cost.

### 6.1. In Crisp Environment

We have taken the value of cost parameters as $p = 20$, $\delta = 5$, $\gamma = 10$, $\gamma_1 = 0.015$, $\lambda = 15$, $C_0 = 3$, $T = 0.9$, $R = 0.2$, $C_h = 10$, $\alpha_d = 0.1$, $\beta_d = 1.8$, $C_d = 11$, and $C_s = 5$. Now solving equation (12), we get

$$
T_1 = 0.4435608
$$

and the value of optimum total

$$
PVTC(T_1) = 286.3966.
$$

### 6.2. In Fuzzy Environment

Here we are taking $p = 20$, $\delta = (1, 5, 9)$, $\gamma = (10, 14, 30)$, $\gamma_1 = 0.015$, $\lambda = (15, 20, 25)$, $C_0 = 3$, $T = 0.9$, $R = 0.2$, $C_h = 10$, $\alpha_d = 0.1$, $\beta_d = 1.8$, $C_d = 11$, and $C_s = 5$. Now solving equation (14) with the help of equations (13), (15), (16), and (17), we get

$$T_1 = 0.4435608$$

and the value of optimum total

$$PVTC(T_1) = 378.4312.$$

## 7. Sensitivity Analysis

Now we are interested to see how the present value of total cost ($PVTC$) changes when some of the values of different parameters are changed.

Firstly, we are considering $\delta$ (the backlogging parameter) and trying to find the effect of the relevant parameter on the total present value of total cost in the crisp case. Here, we are taking other parameter values as they were. Let us take $p = 20$, $\gamma = 10$, $\gamma_1 = 0.015$, $\lambda = 15$, $C_0 = 3$, $T = 0.9$, $R = 0.2$, $C_h = 10$, $\alpha_d = 0.1$, $\beta_d = 1.8$, $C_d = 11$, and $C_s = 5$.

**Table 1.** Sensitivity analysis of the backlogging parameter $\delta$

| The backlogging parameter ($\delta$) | $T_1$ | $PVTC(T_1)$ |
|---|---|---|
| 5 | 0.4435608 | 286.3966 |
| 5.1 | 0.4443857 | 305.0152 |
| 6 | 0.4520748 | 479.9748 |
| 6.2 | 0.4538100 | 520.5907 |
| 6.5 | 0.4564102 | 582.6362 |

From the above Table 1 we clearly see that total PVTC is increases proportionally to the backlogging parameter ($\delta$). From the below chart 1 (see Fig.2) it is also clear that total PVTC is increases proportionally to the backlogging parameter ($\delta$).

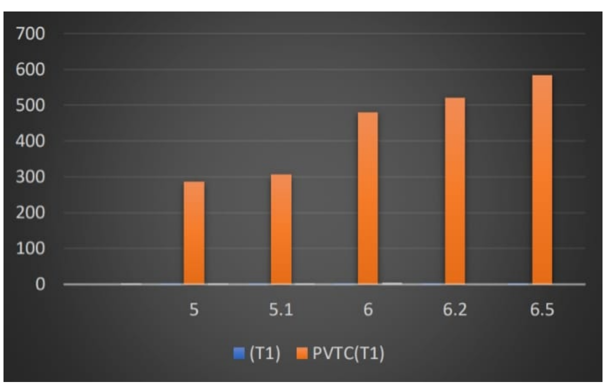


**Figure 2.** Chart 1, which shows the total PVTC is increases proportionally to the backlogging parameter ($\delta$).

Secondly, we are considering $\lambda$ (the price sensitivity parameter) and trying to find the effect of relevant parameter in total present value of total cost in crisp. Here , we are taking other parameters values as it was. Let us take $p = 20$, $\delta = 5$, $\gamma_1 = 0.015$, $\gamma = 10$, $C_0 = 3$, $T = 0.9$, $R = 0.2$, $C_h = 10$, $\alpha_d = 0.1$, $\beta_d = 1.8$, $C_d = 11$, and $C_s = 5$.

**Table 2.** Sensitivity analysis of the price sensitivity parameter $\lambda$

| The price sensitivity parameter ($\lambda$) | $T_1$ | $PVTC(T_1)$ |
|---|---|---|
| 15 | 0.4435608 | 286.3966 |
| 20 | 0.4435608 | 384.3058 |
| 22 | 0.4435608 | 423.4695 |
| 25 | 0.4435608 | 482.2150 |
| 30 | 0.4435608 | 580.1241 |

From the above Table 2 we clearly see that total PVTC is increases proportionally to the price sensitivity parameter ($\lambda$). From the below chart 2 (see Fig. 3) it is also clear that total PVTC is increases proportionally to the price sensitivity parameter ($\lambda$).

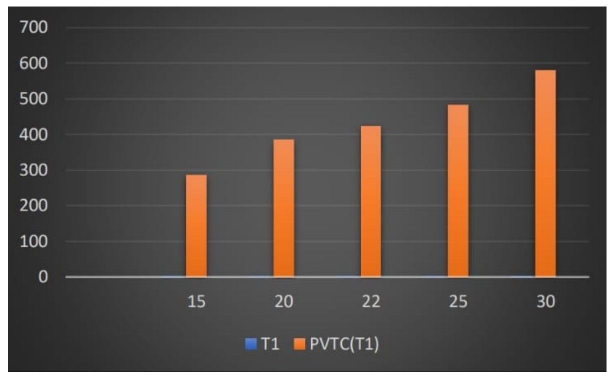


**Figure 3.** Chart 2, which shows the total PVTC is increases proportionally to the price sensitivity parameter ($\lambda$).

Finally, we are considering $\gamma$ (the basic market demand parameter) and trying to find the effect of relevant parameter in total present value of total cost in crisp. Here , we are taking other parameters values as it was. Let us take $p = 20$, $\delta = 5$, $\gamma_1 = 0.015$, $\lambda = 15$, $C_0 = 3$, $T = 0.9$, $R = 0.2$, $C_h = 10$, $\alpha_d = 0.1$, $\beta_d = 1.8$, $C_d = 11$, and $C_s = 5$.

**Table 3.** Sensitivity analysis of the basic market demand parameter $\gamma$

| The basic market demand parameter ($\gamma$) | $T_1$ | $PVTC(T_1)$ |
|---|---|---|
| 5 | 0.4435608 | 291.2921 |
| 10 | 0.4435608 | 286.3966 |
| 20 | 0.4435608 | 276.6057 |
| 30 | 0.4435608 | 266.8148 |
| 40 | 0.4435608 | 257.0239 |

From the above Table 3 we clearly see that total PVTC is inversly proportional to the basic market demand parameter ($\gamma$). From the below chart 3 (see Fig. 4) it is also clear that total PVTC is inversly proportional to the basic market demand parameter ($\gamma$).

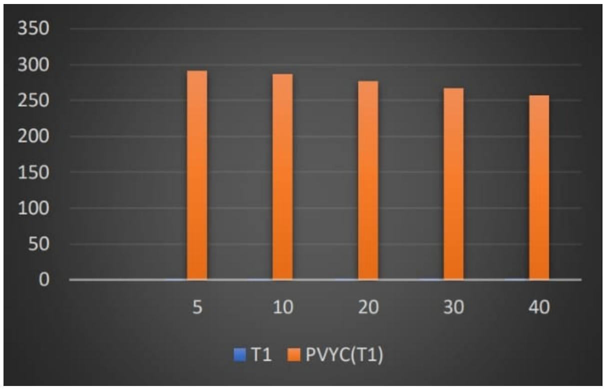


**Figure 4.** Chart 3, presented to show the total PVTC is inversly proportional to the basic market demand parameter ($\gamma$).

## 8. Conclusion

This study develops a comprehensive inventory model for Weibull-deteriorating items under price- and stock-dependent demand in the presence of inflation, time discounting, and partial backlogging of shortages. By integrating fuzzy and intuitionistic fuzzy frameworks into the classical crisp model, the proposed approach effectively captures real-world uncertainty in key parameters such as market demand, price sensitivity, and backlogging behavior. The use of the expected value method enables practical defuzzification and determination of optimal replenishment policies. Numerical illustrations and sensitivity analyses demonstrate that intuitionistic fuzzy models yield more robust and stable solutions compared to crisp and fuzzy counterparts, particularly under high uncertainty. These findings confirm that the proposed model is well suited for managing inventories of short-life-cycle and highly volatile products, such as fashion goods and electronic items, thereby offering valuable managerial insights and improved decision support in uncertain business environments.

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